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IX Conference of Mathematical Modelling in Physics and Engineering

THE JACOBIAN HAVING NON – GENERIC DEGREE

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The subject of this presentation are the leading forms of polynomial mapping of two complex variables in the case when the Jacobian of this mapping has not reached its maximum degree. We consider the mappings having two zeros at infinity.

Let f_m, h_n are the forms of variables X, Y of degrees m and n, respectively ($m \ge n \ge 2$) and the Jacobian of these forms vanishes.

We represent the structure of the forms f_m, h_n .

THEOREM

Let $\mathrm{Jac}(f_m, h_n) = 0$.

Therefore

$$f_m = a\left[(\alpha_1 X + \beta_1 Y)^{p_1} \ldots (\alpha_k X + \beta_k Y)^{p_k}\right]^{\tilde{m}}$$

$$h_n = b\left[(\alpha_1 X + \beta_1 Y)^{p_1} \ldots (\alpha_k X + \beta_k Y)^{p_k}\right]^{\tilde{n}}$$

where $\tilde{m} = m/(m,n)$, $\tilde{n} = n/(m,n)$

$p_1 + \ldots + p_k = (m,n)$

(m,n) *means the greatest common divisor of the numbers m and n*

$a, b, \alpha_i, \beta_i \in C$

$$\det\begin{bmatrix} \alpha_i & \beta_i \\ \alpha_j & \beta_j \end{bmatrix} \ne 0 \quad \textit{for} \quad i \ne j$$

Remark . We can assume that $p_1 \ge \ldots \ge p_k$.

COROLLARIES

1) *Let* $f = f_m + \tilde{f},\ h = h_n + \tilde{h}$, *where* $\det \tilde{f} < m$, $\det \tilde{h} < n$.

If $\mathrm{Jac}(f_m, h_n) = 0$, *then only zeros at infinity of the mapping* (f, h) *are the factors of the form* f_m *or* h_n .

2) *If the numbers m and n are relatively prime and* $\mathrm{Jac}(f_m, h_n) = 0$, *then* $f_m = a(\alpha X + \beta Y)^m$ *and* $h_n = b(\alpha X + \beta Y)^n$.

This means that the mapping (f, h) *has only one zero at infinity.*

3) *Let* $f = f_m + \tilde{f},\ h = h_n + \tilde{h}$, *where* $\det \tilde{f} < m$ *and* $\det \tilde{h} < n$. *Let also* $\mathrm{Jac}(f_m, h_n) = 0$.

If the mapping (f, h) *have two zeros at infinity, then*

1. $f = (XY)^p + \tilde{f}$ *and* $h = (XY)^q + \tilde{h}$, *where* $p \ge q \ge 1$
2. $f = \left(X^k Y^l\right)^p + \tilde{f}$ *and* $h = \left(X^k Y^l\right)^q + \tilde{h}$, *where* $k > l$, *k and l are relatively prime and* $p \ge q \ge 1$.

REMARK

The Jacobians with non-generic degrees occur, among others, when the Jacobian of the polynominal mapping is constant and the co-ordinates of these mappings are not constant polynomials. This case is most interesting and it is considered in the article "Some remarks to the Jacobian Conjecture,,.